

Answers:

$$\begin{aligned}(1) \text{ (a) } A^2 &= \begin{pmatrix} 1 & 5 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} 1 & 5 \\ 4 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 1+20 & 5+10 \\ 4+8 & 20+4 \end{pmatrix} \\ &= \begin{pmatrix} 21 & 15 \\ 12 & 24 \end{pmatrix}\end{aligned}$$

$$\text{(b) } AB = C$$

$$\begin{aligned}\begin{pmatrix} 1 & 5 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} p & 3 \\ q & 4 \end{pmatrix} &= \begin{pmatrix} -3 & 23 \\ 6 & 20 \end{pmatrix} \\ \begin{pmatrix} p+5q & 3+20 \\ 4p+2q & 12+8 \end{pmatrix} &= \begin{pmatrix} -3 & 23 \\ 6 & 20 \end{pmatrix} \\ \begin{pmatrix} p+5q & 23 \\ 4p+2q & 20 \end{pmatrix} &= \begin{pmatrix} -3 & 23 \\ 6 & 20 \end{pmatrix} \\ p+5q &= -3 \\ p &= -5q-3 \\ 4p+2q &= 6 \\ 2p+q &= 3 \\ 2(-5q-3)+q &= 3 \\ -10q-6+q &= 3 \\ -9q &= 9 \\ q &= -1 \\ p &= -5(-1)-3 = 2\end{aligned}$$