

Answers:

$$(1) \text{ (i) } \overrightarrow{CB} = \overrightarrow{AB} - \overrightarrow{AC}$$

$$= b - a$$

$$\overrightarrow{CE} = \frac{2}{3}\overrightarrow{CB} = \frac{2}{3}b - \frac{2}{3}a$$

$$\overrightarrow{AE} = \overrightarrow{AC} + \overrightarrow{CE}$$

$$= a + \frac{2}{3}b - \frac{2}{3}a$$

$$= \frac{2}{3}b + \frac{1}{3}a$$

$$\overrightarrow{AD} = \frac{2}{3}\overrightarrow{AB} = \frac{2}{3}b$$

$$\overrightarrow{DE} = \overrightarrow{AE} - \overrightarrow{AD}$$

$$= \frac{2}{3}b + \frac{1}{3}a - \frac{2}{3}b = \frac{1}{3}a = \frac{1}{3}\overrightarrow{AC}$$

$\therefore DE \parallel AC$

$$(ii) \overrightarrow{AX} = \overrightarrow{AC} + \overrightarrow{CX}$$

$$= a + b - \frac{1}{2}a$$

$$= b + \frac{1}{2}a$$

$$\overrightarrow{AE} = \frac{2}{3}b + \frac{1}{3}a = \frac{2}{3}\left(b + \frac{1}{2}a\right) = \frac{2}{3}\overrightarrow{AX}$$

Therefore points A, E and X are collinear.